Paper 3 Q1.

Q1 a) To escape, an object must have enough kinetic energy that

An object escaping from the surface of a planet must increase its gravitational potential energy to zero (from its current negative value). Its kinetic energy is converted into gravitational potential energy.

Thus $$\frac{1}{2} M V_{esc}^2 = 0 - \left(- \frac{G M_E M}{r} \right)$$

$$\Rightarrow \quad V_{esc}^2 = \frac{2 G M_E}{r}$$

$$\Rightarrow \quad V_{esc} = \sqrt{\frac{2 G M_E}{r}}.$$

b) Because the definition of "escape velocity" is that velocity which would enable the projectile to travel away from the surface indefinitely; never coming back to the ground. Given enough time, then, it could go as far as desired from the planet, its gravitational potential energy becoming arbitrarily close to zero. ~~This increase~~ in gravitational potential energy must have come from a decrease in kinetic energy. We can thus say that the increase in gravitational potential energy must match the decrease in kinetic energy - and since the final kinetic energy can be no lower than zero, this quantity can also be equated to the minimum possible initial kinetic energy.

1 c) (i) Sun's radius $= 7.0 \times 10^8 m$

Density $= 1.4 \times 10^3 kg/m^3$

$M_s = \frac{4}{3}\pi r^3 \times \rho = \frac{4}{3}\pi \times (7 \times 10^8 m)^3 \times 1.4 \times 10^3 kg/m^3$

$= \underline{2.0 \times 10^{30} kg}$.

ii) $V_{esc} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} Nm^2kg^{-2} \times 2.0 \times 10^{30} kg}{7.0 \times 10^8 m}}$

$= \underline{6.2 \times 10^5 ms^{-1}}$.

d) New v_{esc} : since G and M are the same, $v \propto \frac{1}{\sqrt{R}}$.

$\sqrt{\frac{\text{Original radius}}{\text{new radius}}} = \frac{\text{new escape velocity}}{\text{old escape velocity}}$.

$\Rightarrow$ New escape velocity = old escape velocity $\times \sqrt{\frac{R_{old}}{R_{new}}}$

$= 6.2 \times 10^5 ms^{-1} \times \sqrt{\frac{7.0 \times 10^8 m}{1.2 \times 10^4 m}}$

$= \underline{1.5 \times 10^8 ms^{-1}}$ (about $\frac{1}{2}$ lightspeed)

e) Rearrange $V_{ex} = \sqrt{\frac{2GM}{R}}$

$$\Rightarrow V_{ex}^2 = \frac{2GM}{R}$$

$$\Rightarrow R = \frac{2GM}{V_{esc}^2}.$$

For $V_{esc} = 3 \times 10^8 ms^{-1}$

$$R = \frac{2 \times 6.67 \times 10^{-11} Nm^2 kg^{-2} \times 2.0 \times 10^{30} kg}{(3 \times 10^8 ms^{-1})^2}$$

$$= 3.0 \times 10^3 m$$

So for the Sun to become a black hole (on this model, which ignores relativity) it would have to have a radius of about 3km.

f) Kinetic energy = change in gravitational potential energy

$$= \frac{GMm}{R} = \frac{6.67 \times 10^{-11} Nm^2 kg^{-2} \times 2.0 \times 10^{30} kg \times 2.0 \times 10^{-3} kg}{12 \times 10^3 m}$$

$$= \underline{\underline{2.2 \times 10^{13} J}}.$$

g) Total kinetic energy of the gas = $N \frac{1}{2}m\langle v^2\rangle$ where N is the number of molecules in ~~a gas~~ a mol.

Ideal gas equation : $pV = nRT = NkT$.

But we also know : $pV = \frac{1}{3}m\langle v^2\rangle N$ (see syllabus sections 16.8, 16.9).

Thus $NkT = \frac{1}{3}Nm\langle v^2\rangle$

$$\Rightarrow \frac{1}{2}m\langle v^2\rangle = \frac{3}{2}kT \quad \text{(see syllabus 16.9)}.$$

So the total kinetic energy, $\frac{1}{2}Nm\langle v^2\rangle$, will equal $\frac{3}{2}NkT$

where N is the number of particles in 1 mol.

We know that this kinetic energy is 2.2×10^{13} J as calculated in f.

$$\frac{3}{2}NkT = E_k$$

$$\Rightarrow \quad T = \frac{\frac{2}{3}E_k}{Nk} = \frac{\frac{2}{3} \times 2.2 \times 10^{13}\,\text{J}}{6 \times 10^{23}\,\text{mol}^{-1} \times 1.38 \times 10^{-23}\,\text{JK}^{-1}}$$

$$= \underline{\underline{1.8 \times 10^{12}\,\text{K}}}$$

h) Photon energy $\sim kT$

i) $hf = kT \Rightarrow f = \frac{kT}{h} = \frac{1.38 \times 10^{-23} JK^{-1} \times 10^{12} K}{6.63 \times 10^{-34} Js}$.

$= 2.1 \times 10^{23} Hz$.

$\lambda = \frac{c}{f} = \frac{3 \times 10^{8} ms^{-1}}{2.1 \times 10^{23} Hz} = \underline{\underline{1.4 \times 10^{-15} m}}$

ii) This is a gamma ray.

2a) The question here is - could a capacitor provide a steady power source for long enough to make the torch useful?

The power of the torch is $2.5V \times 0.3A = \underline{0.75W}$. ($P = VI$)

To be of any use the torch must last for at least 10 minutes, I'd say, and hopefully a lot longer. The energy stored would be equal to power $\times$ time $= 0.75W \times 600s = \underline{450J}$.

The energy stored in a capacitor is given by $E = \frac{1}{2}QV = \frac{1}{2}CV^2$. A simple model in which the p.d. across the capacitor is the same as that across the bulb (2.5V) would yield:

$$C = \frac{2E}{V^2} = \frac{2 \times 450J}{(2.5V)^2} = 144F.$$

This is a massive capacitance far in excess of any commonly available electronic capacitor (which normally have capacitances measured in millifarads at most). A further difficulty would be that the p.d. across a capacitor drops as it discharges, which would make it difficult to deliver a steady ~~same~~ power to the bulb.

2b) A shower probably requires a temperature of about 50°C, perhaps 30°C warmer than the incoming water. We want to find the electrical power of the shower.

The electrical power input must equal or exceed the thermal power output.
(P_{IN}) (P_{OUT}).

$P_{IN} = VI$ $\qquad P_{OUT} = \frac{\text{Energy delivered}}{\text{time taken}} = \frac{E_{OUT}}{t}$.

E_{OUT} = Mass of water × change in temp × s.h.c
$= m \times \Delta\theta \times$ s.h.c.

$m = \rho \times V$.

Thus $P_{OUT} = \frac{\rho V \Delta\theta \times \text{s.h.c}}{t}$

$\rho = 1000\ kg/m^3 \quad V = 10\ \text{litres} = 10\,dm^3 = 0.01\,m^3. \quad \Delta\theta = 30°C$

$\text{s.h.c} = 4200\ J\,kg^{-1}\,K^{-1} \quad t = 300\,s$.

$P_{OUT} = 4200\,W$

This suggest a minimum current of $\frac{4200W}{230V} \approx \underline{\underline{18\,A}}$,

which tallies with the observation that a 13A fuse is insufficient. In the winter things are likely to be even worse, as the starting temperature of the water will be lower and will require a higher power to raise its temperature. Perhaps some pre-heating by piping the water through a warm indoor area would help.

3a i) The oscillation period is almost exactly 0.50 s.
The frequency is $\frac{1}{T} = 2.0\,Hz$.

ii) Since the ~~amp~~ amplitude of the oscillations continually decreases, you can see that damping is occurring.

b) $\omega = \sqrt{\frac{k}{m}} = 2\pi f$.

$$\Rightarrow \frac{k}{m} = (2\pi f)^2$$

$$\Rightarrow k = m(2\pi f)^2 = 750\,kg \times (2 \times \pi \times 2.0\,Hz)^2$$

$$= 1.2 \times 10^5\,Nm^{-1}$$

But there are 4 wheels, each of which acts as a spring, and they are all in parallel. If we can say they are all identical, the spring constant of each is a quarter of the whole

$$= \underline{\underline{3 \times 10^4\,Nm^{-1}}}.$$

c i) If a periodic force is applied at a frequency close to the natural frequency of oscillation, large amplitude resonant response may occur.

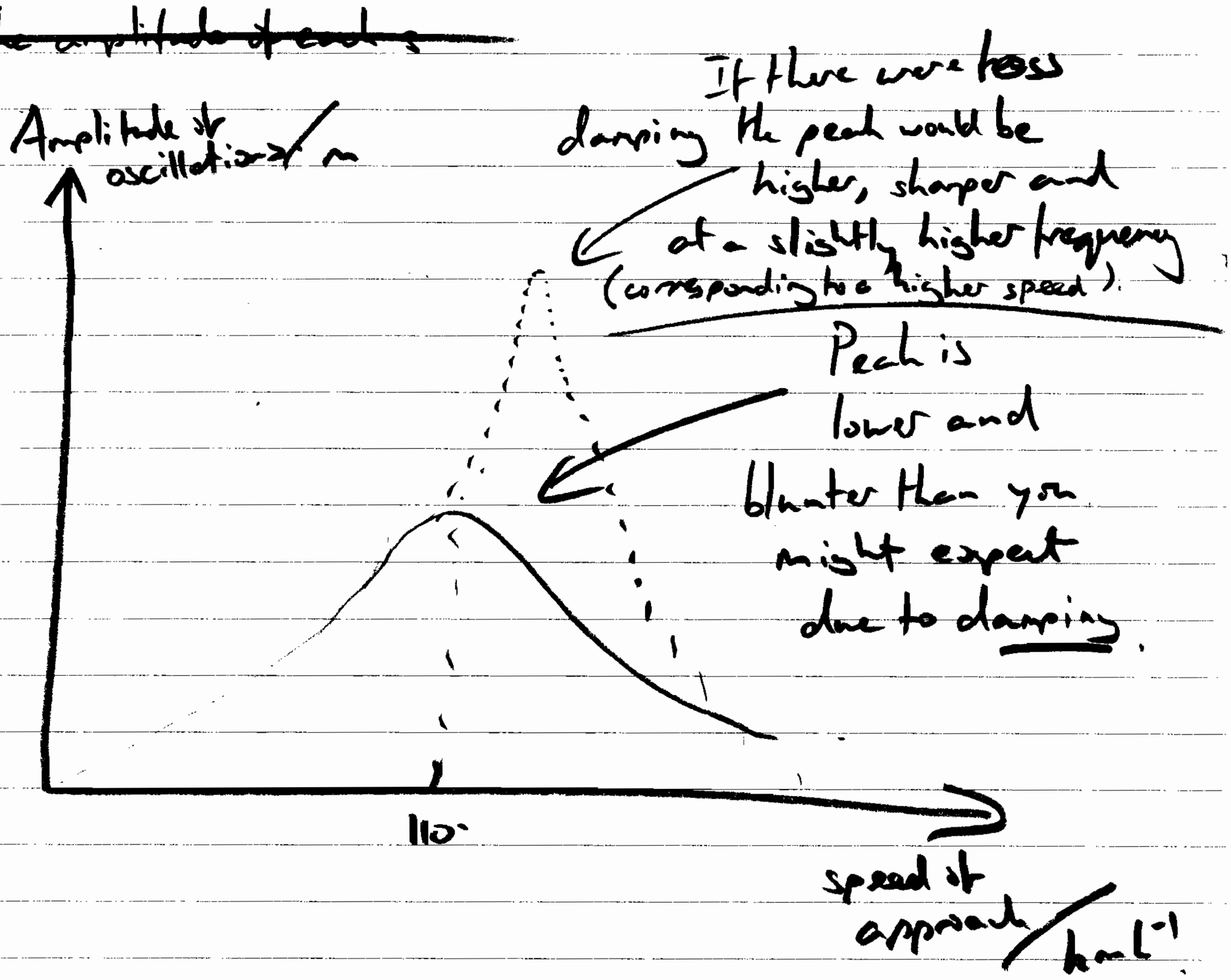
ii) To get a driving force near the 2.0 Hz natural frequency, you must go over a bump every $\frac{1}{2}$ second.
~~So they need~~ $110\,kmh^{-1} = \left(110 \times \frac{1000}{3600}\right)ms^{-1} = 30.6\,ms^{-1}$.
So the bumps should be just over 15 m apart.

Q 3 c i) The oscillation period is a shade below ~~0.5~~ 0.44 s.

~~$f = \frac{1}{t} = \frac{1}{0.44} = 2.3\text{ Hz}$.~~

~~ii) The amplitude of each~~

c iii)



4b i) Light below a certain frequency will not liberate electrons.

ii) If the magnesium ribbon in part c) is illuminated with light of successively longer wavelengths, a point will be reached where the ~~magnesium~~ gold leaf no longer falls. This point will occur when the frequency of the light drops below the threshold.

iii) If a higher intensity source is used above the threshold frequency, electrons will be liberated at a greater rate and the gold leaf will drop faster.

If the light is below the threshold frequency, nothing will happen no matter how great the intensity.

c) i) $E = \frac{hc}{\lambda}$

h : Planck's constant
c : speed of light in a vacuum
λ : wavelength of light.

ii) Rate of photon emission $= \frac{P}{E_\gamma} = \frac{P\lambda}{hc} = \frac{0.50\,W \times 500\times10^{-9}\,m}{6.63\times10^{-34}\,Js \times 3\times10^{8}\,ms^{-1}}$

$= 12.6\times10^{17}$ photons s^{-1} $= 1.3\times10^{18}$ photons/s (to 2 sf).

iii) $4.5\,eV = 4.5 \times 1.6\times10^{-19}\,J = 7.2\times10^{-19}\,J$.
(work function)

Photon energy $= \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\,Js \times 3\times10^{8}\,ms^{-1}}{500\times10^{-9}\,m} = 4.0\times10^{-19}\,J$.

So no, the photoelectric effect cannot occur. The photon energy is less than the work function.

4d) Line spectra show that atoms only absorb and emit light at ~~a~~ certain frequencies. Bohr's model of the atom explains this by saying that electrons in atoms can only have certain energies. As they move from a higher to a lower energy level, they release the difference in energy as a photon of light.

But for this to make sense, we must accept that it is the frequency of the light which is associated with the photon energy rather than the amplitude. We must also accept the idea that light comes in discrete packets or quanta (this is what a photon is) rather than having a purely wave nature. Otherwise there is no reason to connect the energy of the transition with the frequency of the light released.

5i) $E = mc^2$ ii) $E = hf$.

When a nuclear reaction occurs, we find that there is a mass defect - if energy is released in the reaction, the mass of the products will be less than that of the reactants. Einstein's mass/energy equivalence equation relates the two - it is the extra potential energy in the original nucleus that leads to its higher mass. This relationship is quite general and applies to chemical reactions in the same way - it is just that the much lower energies involved mean that the mass defect is undetectable.

The energy of the radiation released in nuclear decay is connected to its frequency by $E = hf$. The energy of nuclear transitions is generally of the order of MeV, so that a typical frequency for the emitted radiation might be:

$$f = \frac{E}{h} = \frac{1\times10^{6}\,\mathrm{eV}\times1.6\times10^{-19}\,\mathrm{J\,eV^{-1}}}{6.63\times10^{-34}\,\mathrm{Js}}$$

$$= 2.4\times10^{20}\,\mathrm{Hz}$$

corresponding to electromagnetic radiation in the gamma region of the spectrum.

5a) The electron and positron feel a force (given by Fleming's left hand rule) at right angles to their motion and to the magnetic field. This curves their tracks.
If there were no energy losses, the tracks would be circles (since the force is at right angles to the velocity). However they are constantly losing their energy by ionizing molecules along their way (otherwise their tracks would not be visible). The radius of the circle ~~the~~ thus decreases, giving a spiral.

b) Q is the positron. R is the electron.

c) The force towards the centre is given by $F = qvB$.

For circular motion, $F = \frac{mv^2}{r}$.

$$\text{Thus } \frac{mv^2}{r} = qvB \Rightarrow v = \frac{qBr}{m}.$$

The positron's path starts with $r \approx 3.0\text{cm}$, while for the electron $r \approx 5.0\text{cm}$.

$$\text{This yields } v_p = \frac{(1.6\times10^{-19}C)\times(2\times10^{-3}T)\times(3\times10^{-2}m)}{9.1\times10^{-31}kg}$$

$$= 1.05\times10^7 ms^{-1} \text{ or } \underline{\underline{1\times10^7 ms^{-1}}} \text{ to 1sf.}$$

$$E_p \approx \tfrac{1}{2}mv^2 = \tfrac{1}{2}\times9.1\times10^{-31}kg\times(1.05\times10^7 ms^{-1})^2$$

$$= 5\times10^{-17}J \approx \underline{\underline{300\,eV}}$$

The electron's velocity V_e is $\frac{5}{3}$ times bigger at $\underline{\underline{1.75\times10^7 ms^{-1}}}$

$$E_e = \tfrac{1}{2}\times9.1\times10^{-31}kg\times(1.75\times10^7 ms^{-1})^2 = 14\times10^{-17}J \approx \underline{\underline{860\,eV}}$$

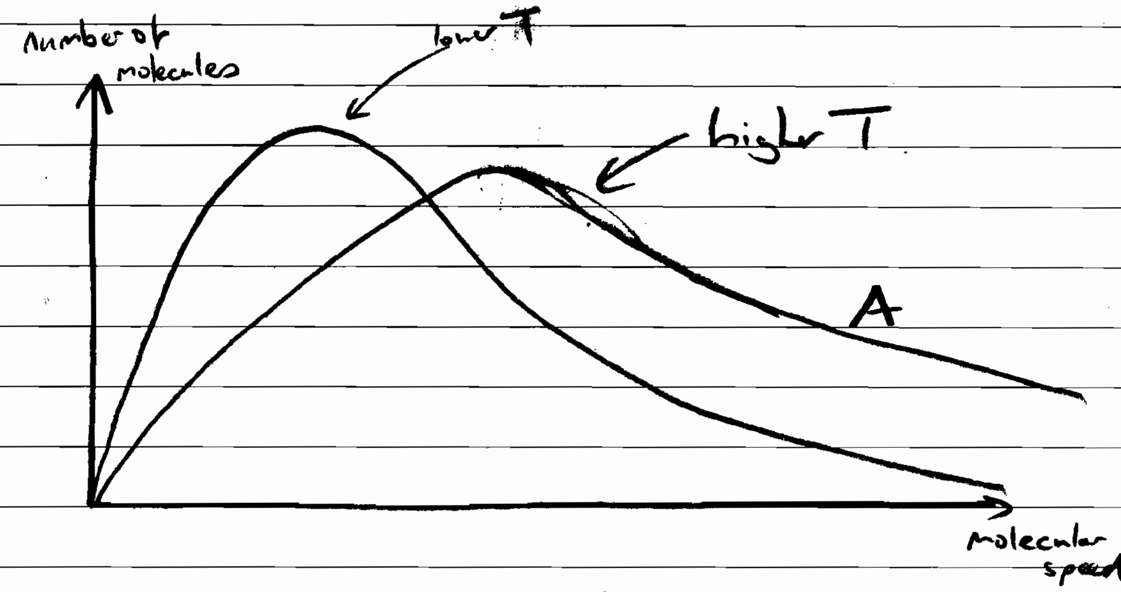
d) $E_\gamma \approx 19\times10^{-17}\,J$

$$E_\gamma = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E_\gamma} = \frac{6.63\times10^{-34}\,Js \times 3\times10^{8}\,ms^{-1}}{1.9\times10^{-16}\,J}$$

$$= 10.5\times10^{-10}\,m \approx \underline{1nm}.$$

In fact this looks more like an X-ray to me.

6a)



b) i) $pV = nRT \Rightarrow n = \dfrac{pV}{RT} = \dfrac{1.0\times10^{4}\,\text{Pa} \times 1.5\times10^{-3}\,\text{m}^{3}}{8.3\,\text{J mol}^{-1}\text{K}^{-1} \times 300\,\text{K}}$

$\Rightarrow n = 0.060\times10^{-1}\,\text{mol} = \underline{6.0\times10^{-3}\,\text{mol}}$

ii) $N = N_A n = 6.0\times10^{-3}\,\text{mol} \times 6.0\times10^{23}\,\text{mol}^{-1} = \underline{\underline{3.6\times10^{21}}}$

iii) $\frac{1}{3}Nm\langle v^2\rangle = pV$

$\Rightarrow \langle v^2\rangle = \dfrac{3pV}{Nm}$

m is the mass of a single molecule $= \dfrac{M_{mol}}{N_A}$.

$\Rightarrow \langle v^2\rangle = \dfrac{3pVN_A}{NM_{mol}}$

$= \dfrac{3\times(1.0\times10^{4}\,\text{Pa})\times(1.5\times10^{-3}\,\text{m}^{3})\times(6.0\times10^{23}\,\text{mol}^{-1})}{(3.6\times10^{21})\times(32\times10^{-3}\,\text{kg mol}^{-1})}$

$= 0.23\times10^{6}\,\text{m}^2\text{s}^{-2} \Rightarrow \sqrt{\langle v^2\rangle} = 0.48\times10^{3}\,\text{m s}^{-1}$

$= \underline{4.8\times10^{2}\,\text{m s}^{-1}}$.

7a i) $45\text{ rpm} = \frac{45}{60}\text{ revs per second} = 2\pi\,\frac{45}{60}\text{ radians per second}$

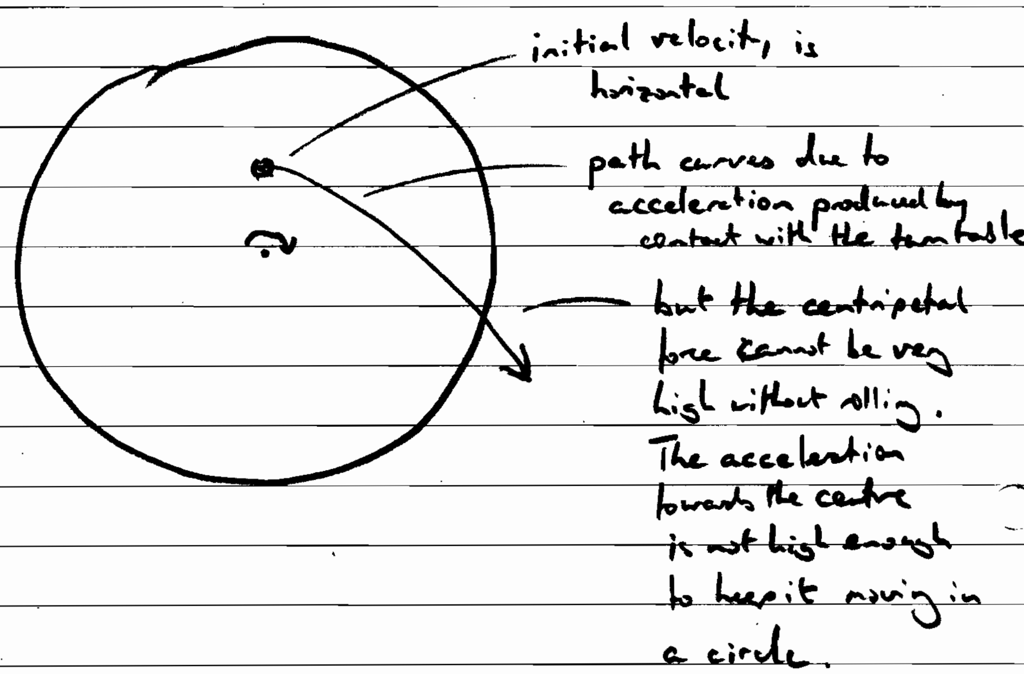
$= 4.7\,(\text{radians})\,s^{-1}$.

ii) $v = r\omega = 6.8\text{ cm} \times 4.7\,(\text{radians})\,s^{-1} = 32\text{ cm/s}$ $(> 30\text{ cm/s})$.

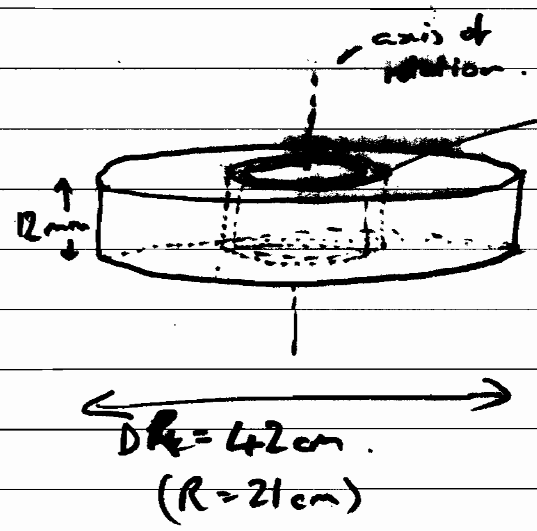
iii) $F = \dfrac{mv^2}{r} = \dfrac{(37 \times 10^{-3}\,\text{kg}) \times (0.32\,\text{m/s})^2}{(6.8 \times 10^{-2}\,\text{m})}$

$= 0.056\,\text{N} = 56\,\text{mN}$ $(> 50\,\text{mN})$.

iv)



b)



Imagine a "Hula-hoop" (TM) of the disc with radius r and thickness δr. Height h is the same as that of the dish (12mm)

To find the total moment of inertia of the disc we must add up all the moments of inertia of the Hula-hoops (TM) which comprise it.

The volume of one Hula-Hoop (TM) is $2\pi r h \delta r$

The mass of one Hula-Hoop (TM) is $\rho 2\pi r h \delta r$.

$\underline{\text{The moment of inertia } \delta I}$ $= mr^2 = 2\pi\rho h r^3 \delta r$.
(of one hoop)

The total $I = \sum \delta I = \sum_{r=0}^{r=R} 2\pi\rho h r^3 \delta r$

or as we let $\delta I \to 0$,

$$I = 2\pi\rho h \int_{r=0}^{r=R} r^3 \, \delta r = 2\pi\rho h \left[\tfrac{1}{4} r^4\right]_0^R$$

$$= \tfrac{1}{2}\pi\rho h R^4 \quad \text{Since } R = \tfrac{1}{2}D$$

$$= \tfrac{1}{2}\pi\rho h \left(\tfrac{1}{2}D\right)^4$$

$$= \tfrac{1}{32}\pi\rho h D^4.$$

ii) $I = \frac{1}{32}\pi \times 2700\,\text{kg m}^{-3} \times (12\times10^{-3}\,\text{m}) \times (0.42\,\text{m})^4$.

$= 0.099\,\text{kg m}^2$ (to 2sf.)

7b iii) The largest contribution to [redacted] inertia comes from the material farthest from the axis. So the best place to drill the holes (assuming that we are allowed to remove only a fixed mass of material) would be on the outer edge of the disc.

c i) $$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{4.71\,\text{rad s}^{-1}}{2.1\,\text{s}} = \underline{2.2\,(\text{rad})\,\text{s}^{-2}}.$$

ii) $$\tau = I\alpha = 0.099\,\text{kg m}^2 \times 2.2\,(\text{rad})\,\text{s}^{-2} = \underline{\underline{0.22\,\text{Nm}}}.$$

iii) The 37g mass (modelling it to be a point mass) would have a moment of inertia mr^2

$$= 0.037\,\text{kg} \times (0.068\,\text{m})^2$$

$$= 1.7 \times 10^{-4}\,\text{kgm}^2.$$

This is less than 0.2% of the moment of inertia of the disc, so it will make little difference to our result if we neglect it.

9a) If a beam of electrons ~~is~~ accelerated in a cathode ray tube (with a potential difference of a few kV) and passes through a crystal, perhaps of graphite, a pattern of maxima and minima (bright and dark rings) appears on the screen.

This interference pattern indicates that the electrons have been diffracted by the crystal - the bright and dark regions are regions of constructive and destructive interference respectively. This indicates that electrons have wave-like properties. (A similar thing can be done with neutrons).

b) The photoelectric effect involves ~~the~~ electrons being removed from a metal surface ~~the~~ when it is illuminated with light. But if the light's frequency is too low, electrons are not liberated, no matter how intense the light is. By contrast, above some threshold frequency, electrons will be liberated even by low intensity light. This indicates that the light is quantised into packets called photons. Only if the energy of a single photon (hf) is enough to liberate an electrons will the effect occur.

c) De Broglie realised that particles display wave-like properties, with a wavelength given by $\lambda = \frac{h}{p}$ where λ is the wavelength, p the momentum and h is Planck's constant. Because electrons have a very small mass, p also tends to be small which gives a value of λ of the order of 10^{-10} m, comparable to the spacing of the layers in a crystal, so that diffraction and interference can be observed. But for a tennis ball, mass is about 10^{30} times greater, so λ is so small that diffraction effects are impossible to observe.

Light waves have frequencies of the order 10^{15} Hz. Radio waves have much lower frequencies, for example about 10^{6} Hz. The photon energy $E = hf$ is thus about 10^{9} times lower for a radio wave than for visible light. Even tiny intensities of ~~the~~ radio radiation will involve huge numbers of photons - the quantization will not be observed.

Both these effects are rooted in the fact that Planck's constant h is very small.

9d) The reality of the situation is something we have to deal with. We are saying that the photon and the electron behave in a way which is in some ways like a wave (the constructive and destructive interference) and in some ways like how we expect a particle to behave (the localization of the photon at a particular point when it interacts). Both models are useful and can be used to predict behaviour. But the critic's question does indicate that neither our intuitive understanding of a wave nor our intuitive understanding of a particle correctly describe the quantum object.

10a) Arrow relative to archer : $u_2 - u_1$

Arrow relative to foot soldier : u_2

Hawk relative to swan : $v_2 - v_1$

Hawk relative to birdwatcher : v_2

Starlight relative to Andromedan : c

Starlight relative to Earthling : c

b) In situation 1 we have simple velocity addition. The speed at which the arrow is [illegible] the speed of the rider gives the final speed of the arrow relative to the foot soldier. The archer fires the arrow at speed $u_2 - u_1$ (relative to him) but the soldier sees it arriving faster, at u_2.

In situation 2 this is not the case. The sound doesn't travel any faster towards the birdwatcher because of the swan's motion. The speed of sound v_2 is relative to the air through which it's travelling. Because the swan is also travelling through the air, the sound only recedes from it at $v_2 - v_1$.

If light were carried in an "aether" we would expect situation 3 to be like situation 2. We would expect the Andromedan to see the light receding at speed $c - x$. But this does not happen. The light leaves at speed c. This is in accord with the principle of relativity - the

Andromedan cannot detect his absolute motion. Light will be emitted at speed c from any inertial frame of reference.

10c) In this thought experiment the observer on the spaceship will correctly observe the flash reaching the front and back detectors simultaneously.

But an external observer will - also correctly - observe the flash reaching the back end before the front end.

This implies that two events which are simultaneous in one frame of reference may not be simultaneous in another. The idea of an absolute time, independent of the frame of reference, is thus not tenable.

d) From the top diagram, we can calculate

$$t = \frac{2W}{c} \Rightarrow W = \frac{1}{2}ct \quad (1)$$

From the lower diagram, we can say

$$x = \frac{1}{2}vt' \text{ (distance moved by ship in half a tick)}$$

$$h = \frac{1}{2}ct' \text{ (distance moved by light in half a tick)}$$

$$W^2 = h^2 - x^2 = \left(\frac{1}{2}ct'\right)^2 - \left(\frac{1}{2}vt'\right)^2 \text{ (Pythagoras)}$$

Substituting from (1), $\left(\frac{1}{2}ct\right)^2 = \left(\frac{1}{2}ct'\right)^2 - \left(\frac{1}{2}vt'\right)^2$

$$\left(\div\left(\tfrac{1}{2}c\right)^2\right) \Rightarrow t^2 = t'^2 - \left(\frac{v}{c}t'\right)^2$$

$$\Rightarrow t^2 = \left(1 - \frac{v^2}{c^2}\right)t'^2 \Rightarrow t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

11a) ~~The~~ Since the wavelength of ~~light is~~ rather small (~ 500 nm) the dimensions of an aperture required to diffract it will also be small.

b) The points of high intensity are points where there is a relatively high probability of the arrival of a photon. The minima are where the probability of photon arrival is low.

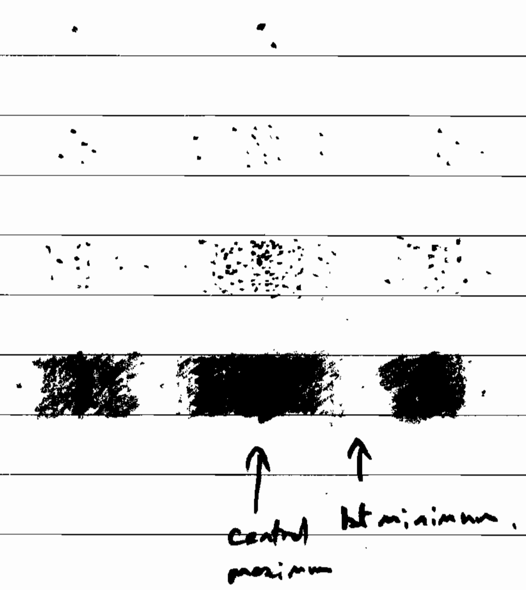
c) If the light is considered as a wave passing through the slit, then each point on the wavefront may be considered as a source of secondary wavelets (Huygen's construction). If the contributions of all these sources are added together, by the principle of superposition, then at some points there will be destructive interference. The amplitude resulting from all these waves added together will be low or zero. At this point there will be a minimum.

d) 10 seconds

100 seconds

1000 seconds

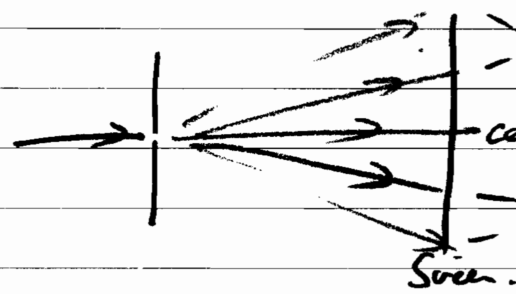
1 year



e) The explanation in c) is based on a wave understanding of light. But the photon-by-photon build-up in d) shows that light is quantised. How then can it be distributed over the slit as required by the wave model? There are several ways of interpreting this - e.g. we can think of a wavefunction ψ whose square $|\psi|^2$ represents the probability density of finding the photon at a particular point.

f) $p = \frac{h}{\lambda}$. An object with a larger momentum will have a smaller de Broglie wavelength.

g) If the slit is made narrower, the diffraction pattern spreads out. The minima and maxima are more widely spaced.

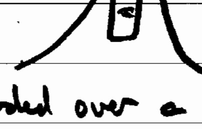


If you regard the light as a stream of photons, it can be reasonably inferred from the diagram above that the photons previously had a pretty definite lateral momentum (zero) but that after passing through the slit they may have a range of lateral momenta, otherwise they couldn't go sideways, as it were. By specifying the lateral position more precisely (by passing the light through the slit) we have generated uncertainty in its lateral momentum. This accords with Heisenberg's uncertainty principle

$$\Delta p \Delta x \sim \frac{h}{2}.$$

h) This principle can also be stated in terms of uncertainty in energy and time:

$$\Delta E \Delta t \sim \frac{h}{2}.$$

A particle in a potential well shaped thus  requires extra energy to get out. But if regarded over a short time period Δt its energy is not well-defined but has uncertainty ΔE. There is thus a small probability that it will be able to escape as it only takes a very small time Δt for it to "tunnel" through the potential barrier. The narrower the barrier, the smaller the time required to tunnel, the larger ΔE and the larger the probability of escape.